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General analysis of the rare $B_c \to D_s^* \ell^+ \ell^-$ decay beyond the standard model

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Abstract. The general analysis of the rare $B_c \to D_s^* \ell^+ \ell^-$ decay is presented by using the most general, model independent effective Hamiltonian. The dependencies of the branching ratios and of the longitudinal, normal and transversal polarization asymmetries for ℓ^- and the combined asymmetries for ℓ^- and ℓ^+ on the new Wilson coefficients are investigated. Our analysis shows that the lepton polarization asymmetries are very sensitive to the scalar and tensor type interactions, which will be very useful in looking for new physics beyond the standard model.

1 Introduction

Rare B meson decays, induced by flavor-changing neutral current (FCNC) $b \to s, d$ transitions, are excellent places to search for new physics, because they appear at the same order as the standard model (SM). In rare B meson decays, the effects of new physics may appear in two different manners: either through the new contributions to the Wilson coefficients existing in the SM or through the new structures in the effective Hamiltonian that are absent in the SM.

There have been many investigations of the new physics through the study of rare radiative, leptonic and semileptonic decays of $B_{u,d,s}$ mesons induced by FCNC transitions of $b \to s, d$[1] since the CLEO observation of $b \to s\gamma$ [2]. The studies will be even more complete if similar decays for B_c are also included.

The study of the B_c meson is by itself quite interesting too, due to its outstanding features [3–5]. It is the lowest bound state of two heavy quarks (b and c) with explicit flavor that can be compared with the charmonium ($c\bar{c}$ bound state) and bottomium ($b\bar{b}$ bound state), which have implicit flavor. The implicit-flavor states decay strongly and electromagnetically, whereas the B_c meson decays weakly. The major difference between the weak decay properties of B_c and $B_{u,d,s}$ is that those of the latter ones are described very well in the framework of the heavy quark limit, which gives some relations between the form factors of the physical process. In the case of the B_c meson, the heavy flavor and spin symmetries must be reconsidered, because both b and c are heavy.

From the experimental side, the running B factories in KEK and SLAC continue to collect data samples and encourage the study of rare B meson decays. It is believed that in future experiments at hadronic colliders, such as the BTeV and LHC-B, most of the rare B_c decays should be accessible.

One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization [7–17]. In this work we present a study of the branching ratio and lepton polarizations in the exclusive $B_c \to D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) decay for a general form of the effective Hamiltonian, including all possible form of interactions in a model independent way without forcing concrete values for the Wilson coefficients corresponding to any specific model.

It is well known that the theoretical study of the inclusive decays is rather easy, but their experimental investigation is difficult. However, for the exclusive decays the situation is contrary to the inclusive case, i.e., their experimental detection is very easy, but the theoretical investigation has its own drawbacks. This is due to the fact that for the description of the exclusive decay the form factors, i.e., the matrix elements of the effective Hamiltonian between initial and final meson states, are needed. This problem is related to the nonperturbative sector of QCD, and it can only be solved in the framework of the nonperturbative approaches.

These matrix elements have been studied in the frameworks of different approaches, such as light front, constituent quark models [5], and the relativistic quark model proposed in [6]. In this work, we will use the weak decay form factors calculated in [6].

The paper is organized as follows. In Sect. 2, we first give the effective Hamiltonian for the quark level process $b \to s\ell^+\ell^-$ and the definitions of the form factors, and then introduce the corresponding matrix element. In Sect. 3, we present the model independent expressions for the longitu-

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dinal, transversal and normal polarizations of the leptons. We also give the combined lepton–antilepton asymmetries. Section 4 is devoted to the numerical analysis and to a discussion of our results.

2 Effective Hamiltonian

In the standard effective Hamiltonian approach, the $B_c \to D_s^* \ell^+\ell^-$ decay is described at the quark level by the $b \to s\ell^+\ell^-$ process, which can be written in terms of twelve model independent four-Fermi interactions, as follows [12]:

$$\begin{aligned}\mathcal{H}_{\rm eff} =& \frac{G\alpha}{\sqrt{2}\pi} V_{ts}V_{tb}^* \\ &\times \Big\{ C_{SL}\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}Lb\bar{\ell}\gamma^\mu\ell + C_{BR}\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}Rb\bar{\ell}\gamma^\mu\ell \\ &+ C_{LL}^{\rm tot}\bar{s}_L\gamma_\mu b_L\bar{\ell}_L\gamma^\mu\ell_L + C_{LR}^{\rm tot}\bar{s}_L\gamma_\mu b_L\bar{\ell}_R\gamma^\mu\ell_R \\ &+ C_{RL}\bar{s}_R\gamma_\mu b_R\bar{\ell}_L\gamma^\mu\ell_L + C_{RR}\bar{s}_R\gamma_\mu b_R\bar{\ell}_R\gamma^\mu\ell_R \\ &+ C_{LRLR}\bar{s}_L b_R\bar{\ell}_L\ell_R + C_{RLLR}\bar{s}_R b_L\bar{\ell}_L\ell_R \\ &+ C_{LRRL}\bar{s}_L b_R\bar{\ell}_R\ell_L + C_{RLRL}\bar{s}_R b_L\bar{\ell}_R\ell_L \\ &+ C_T\bar{s}\sigma_{\mu\nu}b\bar{\ell}\sigma^{\mu\nu}\ell + \mathrm{i}C_{TE}\epsilon^{\mu\nu\alpha\beta}\bar{s}\sigma_{\mu\nu}b\bar{\ell}\sigma_{\alpha\beta}\ell \Big\} ,\end{aligned} \quad (1)$$

where the chiral projection operators L and R in (1) are defined by

$$L = \frac{1-\gamma_5}{2}, \quad R = \frac{1+\gamma_5}{2},$$

and C_X are the coefficients of the four-Fermi interactions. The coefficients C_{SL} and C_{BR} are the nonlocal Fermi interactions that correspond to $-2m_s C_7^{\rm eff}$ and $-2m_b C_7^{\rm eff}$ in the SM, respectively. The following four terms in (1) are the vector type interactions with coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR}. Two of these vector interactions, containing $C_{LL}^{\rm tot}$ and $C_{LR}^{\rm tot}$, already exist in the SM in combinations of the form $(C_9^{\rm eff} - C_{10})$ and $(C_9^{\rm eff} + C_{10})$. Therefore, we write

$$C_{LL}^{\rm tot} = C_9^{\rm eff} - C_{10} + C_{LL} ,$$
$$C_{LR}^{\rm tot} = C_9^{\rm eff} + C_{10} + C_{LR} ,$$

so that $C_{LL}^{\rm tot}$ and $C_{LR}^{\rm tot}$ describe the sum of the contributions from SM and new physics. The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining two terms with the coefficients C_T and C_{TE} describe the tensor type interactions.

After giving the general form of the four-Fermi interaction for the $b \to s\ell^+\ell^-$ transition, we now need to estimate the matrix element for the $B_c \to D_s^*\ell^+\ell^-$ decay. These can be expressed in terms of the invariant form factors as follows:

$$\begin{aligned}&\big\langle D_s^*(p_{D^*},\varepsilon)\big|\bar{s}\gamma_\mu(1\pm\gamma_5)b\big|B_c(p_{B_c})\big\rangle \\ &= -\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{D^*}^\lambda q^\sigma\frac{2V(q^2)}{m_{B_c}+m_{D^*}} \pm \mathrm{i}\varepsilon_\mu^*(m_{B_c}-m_{D^*})A_0(q^2) \\ &\quad \mp \mathrm{i}(p_{B_c}+p_{D^*})_\mu(\varepsilon^* q)\frac{A_+(q^2)}{m_{B_c}+m_{D^*}} \\ &\quad \mp \mathrm{i}q_\mu(\varepsilon^* q)\frac{A_-(q^2)}{m_{B_c}+m_{D^*}} ,\end{aligned} \quad (2)$$

$$\begin{aligned}&\big\langle D_s^*(p_{D^*},\varepsilon)\big|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu(1\pm\gamma_5)b\big|B(p_{B_c})\big\rangle \\ &= 2\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{D^*}^\lambda q^\sigma g(q^2) \\ &\quad \pm \mathrm{i}\big[\varepsilon_\mu^*(m_{B_c}^2-m_{D^*}^2) - (p_{B_c}+p_{D^*})_\mu(\varepsilon^* q)\big]a_0(q^2) \\ &\quad \pm \mathrm{i}(\varepsilon^* q)\left[q_\mu - (p_{B_c}+p_{D^*})_\mu\frac{q^2}{m_{B_c}^2-m_{D^*}^2}\right] \\ &\quad \times \frac{(m_{B_c}^2-m_{D^*}^2)}{q^2}\big(a_+(q^2)-a_0(q^2)\big) ,\end{aligned} \quad (3)$$

$$\begin{aligned}&\big\langle D_s^*(p_{D^*},\varepsilon)\big|\bar{s}\sigma_{\mu\nu}b\big|B(p_{B_c})\big\rangle \\ &= \mathrm{i}\epsilon_{\mu\nu\lambda\sigma}\Big[-g(q^2)\varepsilon^{*\lambda}(p_{B_c}+p_{D^*})^\sigma \\ &\quad + \frac{1}{q^2}\big(m_{B_c}^2-m_{D^*}^2\big)\varepsilon^{*\lambda}q^\sigma\big(g(q^2)-a_0(q^2)\big) \\ &\quad - \frac{2}{q^2}\big(g(q^2)-a_+(q^2)\big)(\varepsilon^* q)p_{D^*}^\lambda q^\sigma\Big] ,\end{aligned} \quad (4)$$

$$\begin{aligned}&\big\langle D_s^*(p_{D^*},\varepsilon)\big|\bar{s}(1\pm\gamma_5)b\big|B(p_{B_c})\big\rangle \\ &= \frac{1}{m_b}\Big[\mp(\varepsilon^* q)(m_{B_c}-m_{D_s^*}) + A_0(q^2) - A_+(q^2) \\ &\quad - \frac{q^2}{m_{B_c}^2-m_{D^*}^2}A_-(q^2)\Big] ,\end{aligned} \quad (5)$$

where $q = p_{B_c} - p_{D_s^*}$ is the momentum transfer, and ε is the polarization vector of D_s^* meson. The matrix element $\langle D_s^*|\bar{s}(1\pm\gamma_5)b|B\rangle$ is calculated by contracting both sides of (2) with q^μ and using the equation of motion.

By using (1)–(5), we can now write the matrix element of the $B_c \to D_s^*\ell^+\ell^-$ decay as

$$\begin{aligned}&\mathcal{M}\big(B_c \to D_s^*\ell^+\ell^-\big) \\ &= \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^*\big\{\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\big[-2A_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{D_s^*}^\lambda q^\sigma \\ &\quad - \mathrm{i}B_1\varepsilon_\mu^* + \mathrm{i}B_2(\varepsilon^* q)(p_{B_c}+p_{D_s^*})_\mu + \mathrm{i}B_3(\varepsilon^* q)q_\mu\big] \\ &\quad + \bar{\ell}\gamma^\mu(1+\gamma_5)\ell\big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{D_s^*}^\lambda q^\sigma - \mathrm{i}D_1\varepsilon_\mu^* \\ &\quad + \mathrm{i}D_2(\varepsilon^* q)(p_{B_c}+p_{D_s^*})_\mu + \mathrm{i}D_3(\varepsilon^* q)q_\mu\big] \\ &\quad + \bar{\ell}(1-\gamma_5)\ell\big[\mathrm{i}B_4(\varepsilon^* q)\big] + \bar{\ell}(1+\gamma_5)\ell\big[\mathrm{i}B_5(\varepsilon^* q)\big] \\ &\quad + 4\bar{\ell}\sigma^{\mu\nu}\ell(\mathrm{i}C_T\epsilon_{\mu\nu\lambda\sigma})\big[-2g\varepsilon^{*\lambda}(p_{B_c}+p_{D_s^*})^\sigma \\ &\quad + B_6\varepsilon^{*\lambda}q^\sigma - B_7(\varepsilon^* q)p_{D_s^*}{}^\lambda q^\sigma\big] \\ &\quad + 16C_{TE}\bar{\ell}\sigma_{\mu\nu}\ell\big[-2g\varepsilon^{*\mu}(p_{B_c}+p_{D_s^*})^\nu \\ &\quad + B_6\varepsilon^{*\mu}q^\nu - B_7(\varepsilon^* q)p_{D_s^*}{}^\mu q^\nu\big]\big\} ,\end{aligned} \quad (6)$$

where

$$A_1 = \big(C_{LL}^{\rm tot} + C_{RL}\big)\frac{V}{m_{B_c}+m_{D_s^*}} - (C_{BR}+C_{SL})\frac{g}{q^2} ,$$

$$\begin{aligned}B_1 =& \big(C_{LL}^{\rm tot} - C_{RL}\big)(m_{B_c}-m_{D_s^*})A_0 \\ &- (C_{BR}-C_{SL})\big(m_{B_c}^2-m_{D_s^*}^2\big)\frac{a_0}{q^2} ,\end{aligned}$$

$$B_2 = \frac{C_{LL}^{\rm tot} - C_{RL}}{m_{B_c}+m_{D_s^*}}A_+ - (C_{BR}-C_{SL})\frac{a_+}{q^2} ,$$

$$\begin{aligned}B_3 =& \big(C_{LL}^{\rm tot} - C_{RL}\big)\frac{A_-}{m_{B_c}+m_{D_s^*}} \\ &+ 2(C_{BR}-C_{SL})(a_+-a_0)\frac{m_{B_c}^2-m_{D_s^*}^2}{q^4} ,\end{aligned}$$

$$
\begin{aligned}
B_4 &= -(C_{LRRL} - C_{RLRL}) \frac{m_{B_c} - m_{D_s^*}}{m_b} \\
&\quad \times \left(A_0 - A_- - \frac{q^2}{m_{B_c}^2 - m_{D_s^*}^2} \right), \\
B_5 &= -(C_{LRLR} - C_{RLLR}) \frac{m_{B_c} - m_{D_s^*}}{m_b} \\
&\quad \times \left(A_0 - A_- - \frac{q^2}{m_{B_c}^2 - m_{D_s^*}^2} \right), \\
B_6 &= \left(m_{B_c}^2 - m_{D_s^*}^2\right) \frac{g - a_0}{q^2}, \\
B_7 &= \frac{2}{q^2}(g - a_0 - a_+), \\
C_1 &= A_1\left(C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}}, C_{RL} \to C_{RR}\right), \\
D_1 &= B_1\left(C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}}, C_{RL} \to C_{RR}\right), \\
D_2 &= B_2\left(C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}}, C_{RL} \to C_{RR}\right), \\
D_3 &= B_3\left(C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}}, C_{RL} \to C_{RR}\right). \qquad (7)
\end{aligned}
$$

3 Lepton polarizations

We now would like to calculate the final lepton polarizations for the $B_c \to D_s^* \ell^+ \ell^-$ decay. To this aim, we will use the convention followed by the earlier works, such as [11, 12], and define the following orthogonal unit vectors: $S_i^{-\mu}$ in the rest frame of ℓ^- and $S_i^{+\mu}$ in the rest frame of ℓ^+, for the polarization of the leptons along the longitudinal ($i = \text{L}$), transverse ($i = \text{T}$) and normal ($i = \text{N}$) directions, by

$$
\begin{aligned}
S_{\text{L}}^{-\mu} &\equiv \left(0, \mathbf{e}_{\text{L}}^-\right) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right), \\
S_{\text{N}}^{-\mu} &\equiv \left(0, \mathbf{e}_{\text{N}}^-\right) = \left(0, \frac{\mathbf{p} \times \mathbf{p}_-}{|\mathbf{p} \times \mathbf{p}_-|}\right), \\
S_{\text{T}}^{-\mu} &\equiv \left(0, \mathbf{e}_{\text{T}}^-\right) = \left(0, \mathbf{e}_{\text{N}}^- \times \mathbf{e}_{\text{L}}^-\right), \\
S_{\text{L}}^{+\mu} &\equiv \left(0, \mathbf{e}_{\text{L}}^+\right) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right), \\
S_{\text{N}}^{+\mu} &\equiv \left(0, \mathbf{e}_{\text{N}}^+\right) = \left(0, \frac{\mathbf{p} \times \mathbf{p}_+}{|\mathbf{p} \times \mathbf{p}_+|}\right), \\
S_{\text{T}}^{+\mu} &\equiv \left(0, \mathbf{e}_{\text{T}}^+\right) = \left(0, \mathbf{e}_{\text{N}}^+ \times \mathbf{e}_{\text{L}}^+\right), \qquad (8)
\end{aligned}
$$

where $\mathbf{p}_\pm$ and $\mathbf{p}$ are the three momenta of $\ell^\pm$ and the D_s^* meson in the center of mass (CM) frame of the $\ell^+\ell^-$ system, respectively. The longitudinal unit vectors S_{L}^- and S_{L}^+ are boosted to the CM frame of $\ell^+\ell^-$ by a Lorentz transformation,

$$
\begin{aligned}
S_{\text{L,CM}}^{-\mu} &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|}\right), \\
S_{\text{L,CM}}^{+\mu} &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, -\frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|}\right), \qquad (9)
\end{aligned}
$$

while vectors of perpendicular directions are not changed by the boost.

The differential decay rate of the $B_c \to D_s^* \ell^+ \ell^-$ decay for any spin direction $\mathbf{n}^\pm$ of the $\ell^\pm$ can be written in the following form:

$$
\frac{\mathrm{d}\Gamma(\mathbf{n}^\pm)}{\mathrm{d}s} = \frac{1}{2}\left(\frac{\mathrm{d}\Gamma}{\mathrm{d}s}\right)_0 \left[1 + \left(P_{\text{L}}^\pm \mathbf{e}_{\text{L}}^\pm + P_{\text{N}}^\pm \mathbf{e}_{\text{N}}^\pm + P_{\text{T}}^\pm \mathbf{e}_{\text{T}}^\pm\right) \cdot \mathbf{n}^\pm\right], \qquad (10)
$$

where $\mathbf{n}^\pm$ is the unit vector in the $\ell^\pm$ rest frame, and $s = q^2/m_{B_c}^2$. Here, the superscripts $^+$ and $^-$ correspond to the ℓ^+ and ℓ^- cases, and the subscript $_0$ corresponds to the unpolarized decay rate, whose explicit form is given by

$$
\begin{aligned}
\left(\frac{\mathrm{d}\Gamma}{\mathrm{d}s}\right)_0 &= \frac{G^2 \alpha^2 m_{B_c}}{2^{14}\pi^5} \left|V_{tb} V_{ts}^*\right|^2 \sqrt{\lambda} v \\
&\times \Big\{ \frac{32}{3} m_{B_c}^4 \lambda \left[\left(m_{B_c}^2 s - m_\ell^2\right)\left(|A_1|^2 + |C_1|^2\right)\right. \\
&+ 6 m_\ell^2 \operatorname{Re}\left(A_1 C_1^*\right)\Big] + 96 m_\ell^2 \operatorname{Re}\left(B_1 D_1^*\right) \\
&- \frac{4}{r} m_{B_c}^2 m_\ell \lambda \operatorname{Re}\left[(B_1 - D_1)\left(B_4^* - B_5^*\right)\right] \\
&+ \frac{8}{r} m_{B_c}^2 m_\ell^2 \lambda \Big(\operatorname{Re}\left[B_1\left(-B_3^* + D_2^* + D_3^*\right)\right] \\
&+ \operatorname{Re}\left[D_1\left(B_2^* + B_3^* - D_3^*\right)\right] - \operatorname{Re}\left(B_4 B_5^*\right)\Big) \\
&+ \frac{4}{r} m_{B_c}^4 m_\ell (1 - r) \lambda \Big(\operatorname{Re}\left[(B_2 - D_2)\left(B_4^* - B_5^*\right)\right] \\
&+ 2 m_\ell \operatorname{Re}\left[(B_2 - D_2)\left(B_3^* - D_3^*\right)\right]\Big) \\
&- \frac{8}{r} m_{B_c}^4 m_\ell^2 \lambda (2 + 2r - s) \operatorname{Re}\left(B_2 D_2^*\right) \\
&+ \frac{4}{r} m_{B_c}^4 m_\ell s \lambda \operatorname{Re}\left[(B_3 - D_3)\left(B_4^* - B_5^*\right)\right] \\
&+ \frac{4}{r} m_{B_c}^4 m_\ell^2 s \lambda |B_3 - D_3|^2 \\
&+ \frac{2}{r} m_{B_c}^2 \left(m_{B_c}^2 s - 2 m_\ell^2\right) \lambda \left(|B_4|^2 + |B_5|^2\right) \\
&- \frac{8}{3rs} m_{B_c}^2 \lambda \left[m_\ell^2 (2 - 2r + s) + m_{B_c}^2 s (1 - r - s)\right] \\
&\times \left[\operatorname{Re}\left(B_1 B_2^*\right) + \operatorname{Re}\left(D_1 D_2^*\right)\right] \\
&+ \frac{4}{3rs} \left[2 m_\ell^2 (\lambda - 6rs) + m_{B_c}^2 s (\lambda + 12rs)\right] \\
&\times \left(|B_1|^2 + |D_1|^2\right) \\
&+ \frac{4}{3rs} m_{B_c}^4 \lambda \left(m_{B_c}^2 s \lambda + m_\ell^2 [2\lambda + 3s(2 + 2r - s)]\right) \\
&\times \left(|B_2|^2 + |D_2|^2\right) \\
&+ \frac{32}{r} m_{B_c}^6 m_\ell \lambda^2 \operatorname{Re}\left[(B_2 + D_2)(B_7 C_{TE})^*\right] \\
&- \frac{32}{r} m_{B_c}^4 m_\ell \lambda (1 - r - s) \\
&\times \left(\operatorname{Re}\left[(B_1 + D_1)(B_7 C_{TE})^*\right] \right. \\
&+ 2 \operatorname{Re}\left[(B_2 + D_2)(B_6 C_{TE})^*\right] \Big) \\
&+ \frac{64}{r} (\lambda + 12rs) m_{B_c}^2 m_\ell \\
&\times \operatorname{Re}\left[(B_1 + D_1)(B_6 C_{TE})^*\right] \\
&+ \frac{256}{3rs} |g|^2 |C_T|^2 m_{B_c}^2 \\
&\times \left(4 m_\ell^2 [\lambda(8r - s) - 12rs(2 + 2r - s)]\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ m_{B_c}^2 s[\lambda(16r+s)+12rs(2+2r-s)]\big) \\
&+ \frac{1024}{3rs}|g|^2|C_{TE}|^2 m_{B_c}^2 \\
&\times \big(8m_\ell^2[\lambda(4r+s)+12rs(2+2r-s)] \\
&+ m_{B_c}^2 s[\lambda(16r+s)+12rs(2+2r-s)]\big) \\
&- \frac{128}{r} m_{B_c}^2 m_\ell[\lambda+12r(1-r)] \\
&\times \mathrm{Re}\left[(B_1+D_1)(gC_{TE})^*\right] \\
&+ \frac{128}{r} m_{B_c}^4 m_\ell \lambda(1+3r-s) \\
&\times \mathrm{Re}\left[(B_2+D_2)(gC_{TE})^*\right] \\
&+ 512 m_{B_c}^4 m_\ell \lambda\, \mathrm{Re}\left[(A_1+C_1)(gC_T)^*\right] \\
&+ \frac{16}{3r} m_{B_c}^2 \\
&\times \big(4(m_{B_c}^2 s+8m_\ell^2)|C_{TE}|^2 + m_{B_c}^2 s v^2 |C_T|^2\big) \\
&\times \big(4(\lambda+12rs)|B_6|^2 + m_{B_c}^4 \lambda^2 |B_7|^2 \\
&- 4m_{B_c}^2(1-r-s)\lambda\, \mathrm{Re}\left(B_6 B_7^*\right) \\
&- 16[\lambda+12r(1-r)]\, \mathrm{Re}\left(g B_6^*\right) \\
&+ 8m_{B_c}^2(1+3r-s)\lambda\, \mathrm{Re}\left(g B_7^*\right)\big)\Big\}\ , \qquad (11)
\end{aligned}
$$

where $\lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $r = m_{D_s^*}^2/m_{B_c}^2$, and $v = \sqrt{1 - \frac{4m_\ell^2}{s m_{B_c}^2}}$ is the lepton velocity.

The polarizations $P_{\mathrm{L}}^{\pm}$, $P_{\mathrm{T}}^{\pm}$ and $P_{\mathrm{N}}^{\pm}$ in (10) are defined by the equation

$$
P_i^{\pm}(q^2) = \frac{\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(\mathbf{n}^{\pm} = \mathbf{e}_i^{\pm}\right) - \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(\mathbf{n}^{\pm} = -\mathbf{e}_i^{\pm}\right)}{\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(\mathbf{n}^{\pm} = \mathbf{e}_i^{\pm}\right) + \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(\mathbf{n}^{\pm} = -\mathbf{e}_i^{\pm}\right)}\,,
$$

for $i = \mathrm{L, N, T}$, i.e., $P_{\mathrm{L}}^{\pm}$ and $P_{\mathrm{T}}^{\pm}$ represent the charged lepton $\ell^{\pm}$ longitudinal and transversal asymmetries in the decay plane, respectively, and $P_{\mathrm{N}}^{\pm}$ is the normal component to both of them. After some lengthy algebra, we get for the longitudinal polarization of the $\ell^{\pm}$

$$
\begin{aligned}
P_{\mathrm{L}}^{\pm} = \frac{4}{\Delta} m_{B_c}^2 v \Big\{ &\mp \frac{1}{3r}\lambda^2 m_{B_c}^4\left[|B_2|^2 - |D_2|^2\right] \\
&+ \frac{1}{r}\lambda m_\ell\, \mathrm{Re}\left[(B_1-D_1)\left(B_4^*+B_5^*\right)\right] \\
&- \frac{1}{r}\lambda m_{B_c}^2 m_\ell(1-r)\, \mathrm{Re}\left[(B_2-D_2)\left(B_4^*+B_5^*\right)\right] \\
&\mp \frac{8}{3}\lambda m_{B_c}^4 s\left[|A_1|^2 - |C_1|^2\right] \\
&- \frac{1}{2r}\lambda m_{B_c}^2 s\left[|B_4|^2 - |B_5|^2\right] \\
&- \frac{1}{r}\lambda m_{B_c}^2 m_\ell s\, \mathrm{Re}\left[(B_3-D_3)\left(B_4^*+B_5^*\right)\right] \\
&\pm \frac{2}{3r}\lambda m_{B_c}^2(1-r-s)\left[\mathrm{Re}\left(B_1B_2^*\right) - \mathrm{Re}\left(D_1D_2^*\right)\right] \\
&\mp \frac{1}{3r}(\lambda+12rs)\left[|B_1|^2 - |D_1|^2\right] \\
&\mp \frac{256}{3}\lambda m_{B_c}^2 m_\ell\big(\mathrm{Re}\left[A_1^*(C_T \mp C_{TE})g\right] \\
&- \mathrm{Re}\left[C_1^*(C_T \pm C_{TE})g\right]\big) \\
&+ \frac{4}{3r}\lambda^2 m_{B_c}^4 m_\ell\big(\mathrm{Re}\left[B_2^*(C_T \mp 4C_{TE})B_7\right] \\
&+ \mathrm{Re}\left[D_2^*(C_T \pm 4C_{TE})B_7\right]\big) \\
&- \frac{8}{3r}\lambda m_{B_c}^2 m_\ell(1-r-s)\big(\mathrm{Re}\left[B_2^*(C_T \mp 4C_{TE})B_6\right] \\
&+ \mathrm{Re}\left[D_2^*(C_T \pm 4C_{TE})B_6\right]\big) \\
&- \frac{4}{3r}\lambda m_{B_c}^2 m_\ell(1-r-s)\big(\mathrm{Re}\left[B_1^*(C_T \mp 4C_{TE})B_7\right] \\
&+ \mathrm{Re}\left[D_1^*(C_T \pm 4C_{TE})B_7\right]\big) \\
&+ \frac{8}{3r}(\lambda+12rs)m_\ell\big(\mathrm{Re}\left[B_1^*(C_T \mp 4C_{TE})B_6\right] \\
&+ \mathrm{Re}\left[D_1^*(C_T \pm 4C_{TE})B_6\right]\big) \\
&- \frac{16}{3r}m_\ell[\lambda+12r(1-r)]\big(\mathrm{Re}\left[B_1^*(C_T \mp 4C_{TE})g\right] \\
&+ \mathrm{Re}\left[D_1^*(C_T \pm 4C_{TE})g\right]\big) \\
&+ \frac{16}{3r}\lambda m_{B_c}^2 m_\ell(1+3r-s)\big(\mathrm{Re}\left[B_2^*(C_T \mp 4C_{TE})g\right] \\
&+ \mathrm{Re}\left[D_2^*(C_T \pm 4C_{TE})g\right]\big) \\
&+ \frac{16}{3r}\lambda^2 m_{B_c}^6 s|B_7|^2\, \mathrm{Re}\left(C_T C_{TE}^*\right) \\
&+ \frac{64}{3r}(\lambda+12rs)m_{B_c}^2 s|B_6|^2\, \mathrm{Re}\left(C_T C_{TE}^*\right) \\
&- \frac{64}{3r}\lambda m_{B_c}^4 s(1-r-s)\, \mathrm{Re}\left(B_6B_7^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
&+ \frac{128}{3r}\lambda m_{B_c}^4 s(1+3r-s)\, \mathrm{Re}\left(B_7 g^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
&- \frac{256}{3r}m_{B_c}^2 s[\lambda+12r(1-r)]\, \mathrm{Re}\left(B_6 g^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
&+ \frac{256}{3r}m_{B_c}^2[\lambda(4r+s)+12r(1-r)^2]|g|^2 \mathrm{Re}\big(C_T C_{TE}^*\big)\Big\}, \qquad (12)
\end{aligned}
$$

where Δ is the term inside the curly brackets of (11).

Similarly, we find for the transverse polarization $P_{\mathrm{T}}^{\pm}$

$$
\begin{aligned}
P_{\mathrm{T}}^{-} = \frac{\pi}{\Delta} m_{B_c}\sqrt{s\lambda}\,\Big\{ &-8m_{B_c}^2 m_\ell\, \mathrm{Re}\left[(A_1+C_1)\left(B_1^*+D_1^*\right)\right] \\
&+ \frac{1}{r}m_{B_c}^2 m_\ell(1+3r-s)\left[\mathrm{Re}\left(B_1D_2^*\right) - \mathrm{Re}\left(B_2D_1^*\right)\right] \\
&+ \frac{1}{rs}m_\ell(1-r-s)\left[|B_1|^2 - |D_1|^2\right] \\
&+ \frac{2}{rs}m_\ell^2(1-r-s)\left[\mathrm{Re}\left(B_1B_5^*\right) - \mathrm{Re}\left(D_1B_4^*\right)\right] \\
&- \frac{1}{r}m_{B_c}^2 m_\ell(1-r-s)\, \mathrm{Re}\left[(B_1+D_1)\left(B_3^*-D_3^*\right)\right] \\
&- \frac{2}{rs}m_{B_c}^2 m_\ell^2\lambda\left[\mathrm{Re}\left(B_2B_5^*\right) - \mathrm{Re}\left(D_2B_4^*\right)\right] \\
&+ \frac{1}{rs}m_{B_c}^4 m_\ell(1-r)\lambda\left[|B_2|^2 - |D_2|^2\right] \\
&+ \frac{1}{r}m_{B_c}^4 m_\ell\lambda\, \mathrm{Re}\left[(B_2+D_2)\left(B_3^*-D_3^*\right)\right] \\
&- \frac{1}{rs}m_{B_c}^2 m_\ell[\lambda+(1-r-s)(1-r)] \\
&\times \left[\mathrm{Re}\left(B_1B_2^*\right) - \mathrm{Re}\left(D_1D_2^*\right)\right] \\
&+ \frac{1}{rs}(1-r-s)\left(2m_\ell^2 - m_{B_c}^2 s\right) \\
&\times \left[\mathrm{Re}\left(B_1B_4^*\right) - \mathrm{Re}\left(D_1B_5^*\right)\right] \\
&+ \frac{1}{rs}m_{B_c}^2\lambda\left(2m_\ell^2 - m_{B_c}^2 s\right)\left[\mathrm{Re}\left(D_2B_5^*\right) - \mathrm{Re}\left(B_2B_4^*\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{16}{rs} \lambda m_{B_c}^2 m_\ell^2 \,\mathrm{Re}\left[(B_1 - D_1)(B_7 C_{TE})^*\right] \\
&+ \frac{16}{rs} \lambda m_{B_c}^4 m_\ell^2 (1-r) \,\mathrm{Re}\left[(B_2 - D_2)(B_7 C_{TE})^*\right] \\
&+ \frac{8}{r} \lambda m_{B_c}^4 m_\ell \,\mathrm{Re}\left[(B_4 - B_5)(B_7 C_{TE})^*\right] \\
&+ \frac{16}{r} \lambda m_{B_c}^4 m_\ell^2 \,\mathrm{Re}\left[(B_3 - D_3)(B_7 C_{TE})^*\right] \\
&+ \frac{32}{rs} m_\ell^2 (1-r-s) \,\mathrm{Re}\left[(B_1 - D_1)(B_6 C_{TE})^*\right] \\
&- \frac{32}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1-r-s) \\
&\times \mathrm{Re}\left[(B_2 - D_2)(B_6 C_{TE})^*\right] \\
&- \frac{16}{r} m_{B_c}^2 m_\ell (1-r-s) \,\mathrm{Re}\left[(B_4 - B_5)(B_6 C_{TE})^*\right] \\
&- \frac{32}{r} m_{B_c}^2 m_\ell^2 (1-r-s) \,\mathrm{Re}\left[(B_3 - D_3)(B_6 C_{TE})^*\right] \\
&- 16 m_{B_c}^2 \big(4 m_\ell^2 \,\mathrm{Re}\left[A_1^*(C_T + 2C_{TE}) B_6\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[A_1^*(C_T - 2C_{TE}) B_6\right]\big) \\
&+ 16 m_{B_c}^2 \big(4 m_\ell^2 \,\mathrm{Re}\left[C_1^*(C_T - 2C_{TE}) B_6\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[C_1^*(C_T + 2C_{TE}) B_6\right]\big) \\
&+ \frac{32}{s} m_{B_c}^2 (1-r) \big(4 m_\ell^2 \,\mathrm{Re}\left[A_1^*(C_T + 2C_{TE}) g\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[A_1^*(C_T - 2C_{TE}) g\right]\big) \\
&- \frac{32}{s} m_{B_c}^2 (1-r) \big(4 m_\ell^2 \,\mathrm{Re}\left[C_1^*(C_T - 2C_{TE}) g\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[C_1^*(C_T + 2C_{TE}) g\right]\big) \\
&+ \frac{64}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1+3r-s) \\
&\times \mathrm{Re}\left[(B_2 - D_2)(g C_{TE})^*\right] \\
&+ \frac{64}{r} m_{B_c}^2 m_\ell^2 (1+3r-s) \,\mathrm{Re}\left[(B_3 - D_3)(g C_{TE})^*\right] \\
&+ \frac{32}{r} m_{B_c}^2 m_\ell (1+3r-s) \,\mathrm{Re}\left[(B_4 - B_5)(g C_{TE})^*\right] \\
&+ \frac{64}{rs} \left[m_{B_c}^2 rs - m_\ell^2 (1+7r-s)\right] \\
&\times \mathrm{Re}\left[(B_1 - D_1)(g C_{TE})^*\right] \\
&- \frac{32}{s} (4 m_\ell^2 + m_{B_c}^2 s) \,\mathrm{Re}\left[(B_1 + D_1)(g C_T)^*\right] \\
&- 2048 m_{B_c}^2 m_\ell \,\mathrm{Re}\left[(C_T g)(B_6 C_{TE})^*\right] \\
&+ \frac{4096}{s} m_{B_c}^2 m_\ell (1-r) |g|^2 \,\mathrm{Re}\left(C_T C_{TE}^*\right) \Bigg\}, \qquad (13)
\end{aligned}
$$

and

$$
\begin{aligned}
P_{\mathrm{T}}^+ = {} & \frac{\pi}{\Delta} m_{B_c} \sqrt{s\lambda} \Big\{ -8 m_{B_c}^2 m_\ell \,\mathrm{Re}\left[(A_1 + C_1)(B_1^* + D_1^*)\right] \\
&- \frac{1}{r} m_{B_c}^2 m_\ell (1+3r-s) \left[\mathrm{Re}\left(B_1 D_2^*\right) - \mathrm{Re}\left(B_2 D_1^*\right)\right] \\
&- \frac{1}{rs} m_\ell (1-r-s) \left[|B_1|^2 - |D_1|^2\right] \\
&+ \frac{1}{rs} \left(2 m_\ell^2 - m_{B_c}^2 s\right)(1-r-s) \\
&\times \left[\mathrm{Re}\left(B_1 B_5^*\right) - \mathrm{Re}\left(D_1 B_4^*\right)\right] \\
&+ \frac{1}{r} m_{B_c}^2 m_\ell (1-r-s) \,\mathrm{Re}\left[(B_1 + D_1)\left(B_3^* - D_3^*\right)\right] \\
&- \frac{1}{rs} m_{B_c}^2 \lambda \left(2 m_\ell^2 - m_{B_c}^2 s\right) \left[\mathrm{Re}\left(B_2 B_5^*\right) - \mathrm{Re}\left(D_2 B_4^*\right)\right] \\
&- \frac{1}{rs} m_{B_c}^4 m_\ell (1-r) \lambda \left[|B_2|^2 - |D_2|^2\right] \\
&- \frac{1}{r} m_{B_c}^4 m_\ell \lambda \,\mathrm{Re}\left[(B_2 + D_2)\left(B_3^* - D_3^*\right)\right] \\
&+ \frac{1}{rs} m_{B_c}^2 m_\ell \left[\lambda + (1-r-s)(1-r)\right] \\
&\times \left[\mathrm{Re}\left(B_1 B_2^*\right) - \mathrm{Re}\left(D_1 D_2^*\right)\right] \\
&+ \frac{2}{rs} m_\ell^2 (1-r-s) \left[\mathrm{Re}\left(B_1 B_4^*\right) - \mathrm{Re}\left(D_1 B_5^*\right)\right] \\
&+ \frac{2}{rs} m_{B_c}^2 m_\ell^2 \lambda \left[\mathrm{Re}\left(D_2 B_5^*\right) - \mathrm{Re}\left(B_2 B_4^*\right)\right] \\
&+ \frac{16}{rs} \lambda m_{B_c}^2 m_\ell^2 \,\mathrm{Re}\left[(B_1 - D_1)(B_7 C_{TE})^*\right] \\
&- \frac{16}{rs} \lambda m_{B_c}^4 m_\ell^2 (1-r) \,\mathrm{Re}\left[(B_2 - D_2)(B_7 C_{TE})^*\right] \\
&- \frac{8}{r} \lambda m_{B_c}^4 m_\ell \,\mathrm{Re}\left[(B_4 - B_5)(B_7 C_{TE})^*\right] \\
&- \frac{16}{r} \lambda m_{B_c}^4 m_\ell^2 \,\mathrm{Re}\left[(B_3 - D_3)(B_7 C_{TE})^*\right] \\
&- \frac{32}{rs} m_\ell^2 (1-r-s) \,\mathrm{Re}\left[(B_1 - D_1)(B_6 C_{TE})^*\right] \\
&+ \frac{32}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1-r-s) \\
&\times \mathrm{Re}\left[(B_2 - D_2)(B_6 C_{TE})^*\right] \\
&+ \frac{16}{r} m_{B_c}^2 m_\ell (1-r-s) \,\mathrm{Re}\left[(B_4 - B_5)(B_6 C_{TE})^*\right] \\
&+ \frac{32}{r} m_{B_c}^2 m_\ell^2 (1-r-s) \,\mathrm{Re}\left[(B_3 - D_3)(B_6 C_{TE})^*\right] \\
&+ 16 m_{B_c}^2 \big(4 m_\ell^2 \,\mathrm{Re}\left[A_1^*(C_T - 2C_{TE}) B_6\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[A_1^*(C_T + 2C_{TE}) B_6\right]\big) \\
&- 16 m_{B_c}^2 \big(4 m_\ell^2 \,\mathrm{Re}\left[C_1^*(C_T + 2C_{TE}) B_6\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[C_1^*(C_T - 2C_{TE}) B_6\right]\big) \\
&- \frac{32}{s} m_{B_c}^2 (1-r) \big(4 m_\ell^2 \,\mathrm{Re}\left[A_1^*(C_T - 2C_{TE}) g\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[A_1^*(C_T + 2C_{TE}) g\right]\big) \\
&+ \frac{32}{s} m_{B_c}^2 (1-r) \big(4 m_\ell^2 \,\mathrm{Re}\left[C_1^*(C_T + 2C_{TE}) g\right] \\
&- m_{B_c}^2 s \,\mathrm{Re}\left[C_1^*(C_T - 2C_{TE}) g\right]\big) \\
&- \frac{64}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1+3r-s) \\
&\times \mathrm{Re}\left[(B_2 - D_2)(g C_{TE})^*\right] \\
&- \frac{64}{r} m_{B_c}^2 m_\ell^2 (1+3r-s) \,\mathrm{Re}\left[(B_3 - D_3)(g C_{TE})^*\right] \\
&- \frac{32}{r} m_{B_c}^2 m_\ell (1+3r-s) \,\mathrm{Re}\left[(B_4 - B_5)(g C_{TE})^*\right] \\
&- \frac{64}{rs} \left[m_{B_c}^2 rs - m_\ell^2 (1+7r-s)\right] \\
&\times \mathrm{Re}\left[(B_1 - D_1)(g C_{TE})^*\right] \\
&- \frac{32}{s} \left(4 m_\ell^2 + m_{B_c}^2 s\right) \mathrm{Re}\left[(B_1 + D_1)(g C_T)^*\right] \\
&- 2048 m_{B_c}^2 m_\ell \,\mathrm{Re}\left[(C_T g)(B_6 C_{TE})^*\right] \\
&+ \frac{4096}{s} m_{B_c}^2 m_\ell (1-r) |g|^2 \,\mathrm{Re}(C_T C_{TE}^*) \Big\}. \qquad (14)
\end{aligned}
$$

Finally, for the normal asymmetries we get

$$
\begin{aligned}
P_{\rm N}^- = \frac{1}{\Delta}\pi v m_{B_c}^3 \sqrt{s\lambda}\, \Big\{ & 8m_\ell \,\mathrm{Im}\left[\left(B_1^* C_1\right) + \left(A_1^* D_1\right)\right] \\
& - \frac{1}{r} m_{B_c}^2 \lambda \,\mathrm{Im}\left[\left(B_2^* B_4\right) + \left(D_2^* B_5\right)\right] \\
& + \frac{1}{r} m_{B_c}^2 m_\ell \lambda \,\mathrm{Im}\left[(B_2 - D_2)\left(B_3^* - D_3^*\right)\right] \\
& - \frac{1}{r} m_\ell (1+3r-s) \,\mathrm{Im}\left[(B_1 - D_1)\left(B_2^* - D_2^*\right)\right] \\
& + \frac{1}{r}(1-r-s) \,\mathrm{Im}\left[\left(B_1^* B_4\right) + \left(D_1^* B_5\right)\right] \\
& - \frac{1}{r} m_\ell (1-r-s) \,\mathrm{Im}\left[(B_1 - D_1)\left(B_3^* - D_3^*\right)\right] \\
& - \frac{8}{r} m_{B_c}^2 m_\ell \lambda \,\mathrm{Im}\left[(B_4 + B_5)(B_7 C_{TE})^*\right] \\
& + \frac{16}{r} m_\ell (1-r-s) \,\mathrm{Im}\left[(B_4 + B_5)(B_6 C_{TE})^*\right] \\
& - \frac{32}{r} m_\ell (1+3r-s) \,\mathrm{Im}\left[(B_4 + B_5)(g C_{TE})^*\right] \\
& - 16 m_{B_c}^2 s \big(\mathrm{Im}\left[A_1^* (C_T - 2C_{TE}) B_6\right] \\
& + \mathrm{Im}\left[C_1^* (C_T + 2C_{TE}) B_6\right]\big) \\
& + 32 m_{B_c}^2 (1-r) \big(\mathrm{Im}\left[A_1^* (C_T - 2C_{TE}) g\right] \\
& + \mathrm{Im}\left[C_1^* (C_T + 2C_{TE}) g\right]\big) \\
& + 32 \big(\mathrm{Im}\left[B_1^* (C_T - 2C_{TE}) g\right] \\
& - \mathrm{Im}\left[D_1^* (C_T + 2C_{TE}) g\right]\big) \\
& + 512 m_\ell \left(|C_T|^2 - 4|C_{TE}|^2\right) \mathrm{Im}\left(B_6^* g\right) \Big\} , \qquad (15)
\end{aligned}
$$

and

$$
\begin{aligned}
P_{\rm N}^+ = \frac{1}{\Delta}\pi v m_{B_c}^3 \sqrt{s\lambda}\, \Big\{ & -8m_\ell \,\mathrm{Im}\left[\left(B_1^* C_1\right) + \left(A_1^* D_1\right)\right] \\
& + \frac{1}{r} m_{B_c}^2 \lambda \,\mathrm{Im}\left[\left(B_2^* B_5\right) + \left(D_2^* B_4\right)\right] \\
& + \frac{1}{r} m_{B_c}^2 m_\ell \lambda \,\mathrm{Im}\left[(B_2 - D_2)\left(B_3^* - D_3^*\right)\right] \\
& - \frac{1}{r} m_\ell (1+3r-s) \,\mathrm{Im}\left[(B_1 - D_1)\left(B_2^* - D_2^*\right)\right] \\
& - \frac{1}{r}(1-r-s) \,\mathrm{Im}\left[\left(B_1^* B_5\right) + \left(D_1^* B_4\right)\right] \\
& - \frac{1}{r} m_\ell (1-r-s) \,\mathrm{Im}\left[(B_1 - D_1)\left(B_3^* - D_3^*\right)\right] \\
& + \frac{8}{r} m_{B_c}^2 m_\ell \lambda \,\mathrm{Im}\left[(B_4 + B_5)(B_7 C_{TE})^*\right] \\
& - \frac{16}{r} m_\ell (1-r-s) \,\mathrm{Im}\left[(B_4 + B_5)(B_6 C_{TE})^*\right] \\
& + \frac{32}{r} m_\ell (1+3r-s) \,\mathrm{Im}\left[(B_4 + B_5)(g C_{TE})^*\right] \\
& - 16 m_{B_c}^2 s \big(\mathrm{Im}\left[A_1^* (C_T + 2C_{TE}) B_6\right] \\
& + \mathrm{Im}\left[C_1^* (C_T - 2C_{TE}) B_6\right]\big) \\
& + 32 m_{B_c}^2 (1-r) \big(\mathrm{Im}\left[A_1^* (C_T + 2C_{TE}) g\right] \\
& + \mathrm{Im}\left[C_1^* (C_T - 2C_{TE}) g\right]\big) \\
& - 32 \big(\mathrm{Im}\left[B_1^* (C_T + 2C_{TE}) g\right] \\
& - \mathrm{Im}\left[D_1^* (C_T - 2C_{TE}) g\right]\big) \\
& + 512 m_\ell \left(|C_T|^2 - 4|C_{TE}|^2\right) \mathrm{Im}\left(B_6^* g\right) \Big\} . \qquad (16)
\end{aligned}
$$

From (12)–(16), we observe that for longitudinal and normal polarizations, the difference between the ℓ^+ and ℓ^- lepton asymmetries results from the scalar and tensor type interactions. A similar situation occurs for transverse polarization asymmetries in the $m_\ell \to 0$ limit. From this, we can conclude that the experimental study of these quantities may provide essential information about new physics.

Another source of useful information about new physics can be a combined analysis of the lepton and antilepton polarizations, since in the SM $P_{\rm L}^- + P_{\rm L}^+ = 0$, $P_{\rm N}^- + P_{\rm N}^+ = 0$ and $P_{\rm T}^- - P_{\rm T}^+ \approx 0$ [12]. Using (12)–(16), we get

$$
\begin{aligned}
P_{\rm L}^- + P_{\rm L}^+ = \frac{4}{\Delta} m_{B_c}^2 v \Big\{ & \frac{2}{r} m_\ell \lambda \,\mathrm{Re}\left[(B_1 - D_1)\left(B_4^* + B_5^*\right)\right] \\
& - \frac{2}{r} m_{B_c}^2 m_\ell \lambda (1-r) \,\mathrm{Re}\left[(B_2 - D_2)\left(B_4^* + B_5^*\right)\right] \\
& - \frac{1}{r} m_{B_c}^2 s\lambda \left(|B_4|^2 - |B_5|^2\right) \\
& - \frac{2}{r} m_{B_c}^2 m_\ell s\lambda \,\mathrm{Re}\left[(B_3 - D_3)\left(B_4^* + B_5^*\right)\right] \\
& + \frac{8}{3r} m_{B_c}^4 m_\ell \lambda^2 \,\mathrm{Re}\left[(B_2 + D_2)(B_7 C_T)^*\right] \\
& + \frac{32}{3r} m_{B_c}^6 s\lambda^2 |B_7|^2 \,\mathrm{Re}\left(C_T C_{TE}^*\right) \\
& - \frac{8}{3r} m_{B_c}^2 m_\ell \lambda (1-r-s) \\
& \times \mathrm{Re}\left[(B_1 + D_1)(B_7 C_T)^*\right] \\
& - \frac{16}{3r} m_{B_c}^2 m_\ell \lambda (1-r-s) \\
& \times \mathrm{Re}\left[(B_2 + D_2)(B_6 C_T)^*\right] \\
& - \frac{128}{3r} m_{B_c}^4 s\lambda (1-r-s) \\
& \times \mathrm{Re}\left(B_6 B_7^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
& + \frac{16}{3r} m_\ell (\lambda + 12rs) \,\mathrm{Re}\left[(B_1 + D_1)(B_6 C_T)^*\right] \\
& + \frac{128}{3r} m_{B_c}^2 s(\lambda + 12rs) |B_6|^2 \,\mathrm{Re}\left(C_T C_{TE}^*\right) \\
& + \frac{512}{3r} m_{B_c}^2 [\lambda(4r+s) + 12r(1-r)^2] |g|^2 \\
& \times \mathrm{Re}\left(C_T C_{TE}^*\right) \\
& - \frac{512}{3r} m_{B_c}^2 s[\lambda + 12r(1-r)] \\
& \times \mathrm{Re}\left(g B_6^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
& + \frac{256}{3r} m_{B_c}^4 s\lambda (1+3r-s) \\
& \times \mathrm{Re}\left(g B_7^*\right) \mathrm{Re}\left(C_T C_{TE}^*\right) \\
& + \frac{512}{3} m_{B_c}^2 m_\ell \lambda \,\mathrm{Re}\left[(A_1 + C_1)(g C_{TE})^*\right] \\
& - \frac{32}{3r} m_\ell [\lambda + 12r(1-r)] \,\mathrm{Re}\left[(B_1 + D_1)(g C_T)^*\right] \\
& + \frac{32}{3r} m_{B_c}^2 m_\ell \lambda (1+3r-s) \\
& \times \mathrm{Re}\left[(B_2 + D_2)(g C_T)^*\right] \Big\} . \qquad (17)
\end{aligned}
$$

For the case of transverse polarization, it is the difference of the lepton and antilepton polarizations that is relevant, and it can be calculated from (13) and (14):

$$\begin{aligned}
P_{\rm T}^- - P_{\rm T}^+ = \frac{\pi}{\Delta} m_{B_c} \sqrt{s\lambda} \Big\{ & \frac{2}{rs} m_{B_c}^4 m_\ell (1-r)\lambda \big[|B_2|^2 - |D_2|^2\big] \\
& + \frac{1}{r} m_{B_c}^4 \lambda \,\mathrm{Re}\left[(B_2+D_2)\big(B_4^* - B_5^*\big)\right] \\
& + \frac{2}{r} m_{B_c}^4 m_\ell \lambda \,\mathrm{Re}\left[(B_2+D_2)\big(B_3^* - D_3^*\big)\right] \\
& + \frac{2}{r} m_{B_c}^2 m_\ell (1+3r-s) \\
& \times \left[\mathrm{Re}\left(B_1 D_2^*\right) - \mathrm{Re}\left(B_2 D_1^*\right)\right] \\
& + \frac{2}{rs} m_\ell (1-r-s) \big[|B_1|^2 - |D_1|^2\big] \\
& - \frac{1}{r} m_{B_c}^2 (1-r-s)\,\mathrm{Re}\left[(B_1+D_1)\big(B_4^* - B_5^*\big)\right] \\
& - \frac{2}{r} m_{B_c}^2 m_\ell (1-r-s) \\
& \times \mathrm{Re}\left[(B_1+D_1)\big(B_3^* - D_3^*\big)\right] \\
& - \frac{2}{rs} m_{B_c}^2 m_\ell [\lambda + (1-r)(1-r-s)] \\
& \times \left[\mathrm{Re}\left(B_1 B_2^*\right) - \mathrm{Re}\left(D_1 D_2^*\right)\right] \\
& - \frac{32}{rs} m_{B_c}^2 m_\ell^2 \lambda \,\mathrm{Re}\left[(B_1 - D_1)(B_7 C_{TE})^*\right] \\
& + \frac{32}{rs} m_{B_c}^4 m_\ell^2 \lambda (1-r)\,\mathrm{Re}\left[(B_2 - D_2)(B_7 C_{TE})^*\right] \\
& + \frac{16}{r} m_{B_c}^4 m_\ell \lambda \,\mathrm{Re}\left[(B_4 - B_5)(B_7 C_{TE})^*\right] \\
& + \frac{32}{r} m_{B_c}^4 m_\ell^2 \lambda \,\mathrm{Re}\left[(B_3 - D_3)(B_7 C_{TE})^*\right] \\
& + \frac{64}{rs} m_\ell^2 (1-r-s)\,\mathrm{Re}\left[(B_1 - D_1)(B_6 C_{TE})^*\right] \\
& - \frac{64}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1-r-s) \\
& \times \mathrm{Re}\left[(B_2 - D_2)(B_6 C_{TE})^*\right] \\
& - \frac{32}{r} m_{B_c}^2 m_\ell (1-r-s) \\
& \times \mathrm{Re}\left[(B_4 - B_5)(B_6 C_{TE})^*\right] \\
& - \frac{64}{r} m_{B_c}^2 m_\ell^2 (1-r-s) \\
& \times \mathrm{Re}\left[(B_3 - D_3)(B_6 C_{TE})^*\right] \\
& + 32 m_{B_c}^4 s v^2 \,\mathrm{Re}\left[(A_1 - C_1)(B_6 C_T)^*\right] \\
& + \frac{64}{r} m_{B_c}^2 m_\ell (1+3r-s) \\
& \times \mathrm{Re}\left[(B_4 - B_5)(g C_{TE})^*\right] \\
& - 64 m_{B_c}^4 (1-r) v^2 \,\mathrm{Re}\left[(A_1 - C_1)(g C_T)^*\right] \\
& + \frac{128}{rs} [m_{B_c}^2 rs - m_\ell^2 (1+7r-s)] \\
& \times \mathrm{Re}\left[(B_1 - D_1)(g C_{TE})^*\right] \\
& + \frac{128}{rs} m_{B_c}^2 m_\ell^2 (1-r)(1+3r-s) \\
& \times \mathrm{Re}\left[(B_2 - D_2)(g C_{TE})^*\right] \\
& + \frac{128}{r} m_{B_c}^2 m_\ell^2 (1+3r-s) \\
& \times \mathrm{Re}\left[(B_3 - D_3)(g C_{TE})^*\right] \Big\} \,. \qquad (18)
\end{aligned}$$

In the same manner, it follows from (15) and (16) that

$$\begin{aligned}
P_{\rm N}^- + P_{\rm N}^+ = \frac{1}{\Delta} \pi v m_{B_c}^3 \sqrt{s\lambda} & \\
\times \Big\{ & -\frac{2}{r} m_\ell (1+3r-s)\,\mathrm{Im}\big[(B_1 - D_1)\big(B_2^* - D_2^*\big)\big] \\
& - \frac{2}{r} m_\ell (1-r-s)\,\mathrm{Im}\left[(B_1 - D_1)\big(B_3^* - D_3^*\big)\right] \\
& - \frac{1}{r} (1-r-s)\,\mathrm{Im}\left[(B_1 - D_1)\big(B_4^* - B_5^*\big)\right] \\
& + \frac{2}{r} m_{B_c}^2 m_\ell \lambda \,\mathrm{Im}\left[(B_2 - D_2)\big(B_3^* - D_3^*\big)\right] \\
& + \frac{1}{r} m_{B_c}^2 \lambda \,\mathrm{Im}\left[(B_2 - D_2)\big(B_4^* - B_5^*\big)\right] \\
& + 32 m_{B_c}^2 s \,\mathrm{Im}\left[(A_1 + C_1)(B_6 C_T)^*\right] \\
& + 1024 m_\ell \big(|C_T|^2 - |4 C_{TE}|^2\big)\,\mathrm{Im}\left(B_6^* g\right) \\
& - 64 m_{B_c}^2 (1-r)\,\mathrm{Im}\left[(A_1 + C_1)(g C_T)^*\right] \\
& + 128\,\mathrm{Im}\left[(B_1 + D_1)(g C_{TE})^*\right] \Big\} \,. \qquad (19)
\end{aligned}$$

It can be seen from (17) that in $P_{\rm L}^- + P_{\rm L}^+$ the terms containing the SM contribution, i.e., terms containing C_{BR}, C_{SL}, $C_{LL}^{\rm tot}$ and $C_{LR}^{\rm tot}$ completely cancel. For this reason, a measurement of the nonzero value of $P_{\rm L}^- + P_{\rm L}^+$ in future experiments may be an indication of the discovery of new physics beyond the SM.

Before going into the details of our numerical analysis we like to note a final point about the numerical calculations of the polarization asymmetries. As seen from (12)–(19), all the expressions of the lepton polarizations depend on both $s = q^2/m_{B_c}^2$ and the new Wilson coefficients. However, it may experimentally be easier to study the dependence of the polarizations of each lepton on the new Wilson coefficients only. For this reason we eliminate the s dependence by considering their averaged forms over the allowed kinematical region. The averaged lepton polarizations are defined by

$$\langle P_i \rangle = \frac{\displaystyle\int_{(2m_\ell/m_{B_c})^2}^{(1-m_{D_s^*}/m_{B_c})^2} P_i \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}s}\,\mathrm{d}s}{\displaystyle\int_{(2m_\ell/m_{B_c})^2}^{(1-m_{D_s^*}/m_{B_c})^2} \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}s}\,\mathrm{d}s} \,. \qquad (20)$$

4 Numerical analysis and discussion

We here present our numerical analysis of the branching ratios and the averaged polarization asymmetries $\langle P_{\rm L}^- \rangle$, $\langle P_{\rm T}^- \rangle$ and $\langle P_{\rm N}^- \rangle$ of ℓ^- for the $B_c \to D_s^* \ell^+ \ell^-$ decays with $\ell = \mu, \tau$, as well as the lepton–antilepton combined asymmetries $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$, $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ and $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$. We first give the input parameters used in our numerical analysis:

$$m_{B_c} = 6.50\,\mathrm{GeV}\,, \qquad m_{D_s^*} = 2.112\,\mathrm{GeV}\,,$$
$$m_b = 4.8\,\mathrm{GeV}\,, \qquad m_\mu = 0.105\,\mathrm{GeV}\,,$$
$$m_\tau = 1.77\,\mathrm{GeV}\,,$$

$$\left|V_{tb}V_{ts}^*\right| = 0.0385\,, \qquad \alpha^{-1} = 129\,,$$
$$G_{\rm F} = 1.17\times 10^{-5}\,{\rm GeV}^{-2}\,, \qquad \tau_{B_c} = 0.46\times 10^{-12}\,{\rm s}\,. \tag{21}$$

The values of the individual Wilson coefficients that appear in the SM are listed in Table 1. The values for the mass and the lifetime of the B_c meson given above in (21) were reported by the CDF Collaboration [20, 21]. Recently, CDF quoted a new value of $m_{B_c} = 6.2857 \pm 0.0053 \pm 0.0012\,{\rm GeV}$ [22]. Also, D0 has observed B_c and reported the preliminary results $m_{B_c} = 5.95^{+0.14}_{-0.13} \pm 0.34\,{\rm GeV}$ and $\tau_{B_c} = 0.45^{+0.12}_{-0.10} \pm 0.12$ [23]. However, we observed that our numerical results are not sensitive to the numerical values of m_{B_c} within 3–5%.

We note that the value of the Wilson coefficient $C_9^{\rm eff}$ in Table 1 corresponds only to the short-distance contributions. $C_9^{\rm eff}$ also receives long-distance contributions due to the conversion of the real $\bar{c}c$ into a lepton pair $\ell^+\ell^-$, and they are usually absorbed into a redefinition of the short-distance Wilson coefficients:

$$C_9^{\rm eff}(\mu) = C_9(\mu) + Y(\mu)\,, \tag{22}$$

where

$$\begin{aligned} Y(\mu) = {} & Y_{\rm reson} + h(y,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) \\ & + C_4(\mu) + 3C_5(\mu) + C_6(\mu)] \\ & - \frac{1}{2}h(1,s)(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ & - \frac{1}{2}h(0,s)[C_3(\mu) + 3C_4(\mu)] \\ & + \frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))\,, \end{aligned} \tag{23}$$

with $y = m_c/m_b$, and the functions $h(y,s)$ arise from the one loop contributions of the four quark operators $O_1, \ldots, O_6$ and their explicit forms can be found in [24–26]. It is possible to parametrize the resonance $\bar{c}c$ contribution $Y_{\rm reson}(s)$ in (23) using a Breit–Wigner shape with normalizations fixed by the data given by [27]:

$$\begin{aligned} Y_{\rm reson}(s) = {} & -\frac{3}{\alpha_{\rm em}^2}\kappa \sum_{V_i=\psi_i} \frac{\pi \Gamma\left(V_i \to \ell^+\ell^-\right) m_{V_i}}{s m_{B_c}^2 - m_{V_i} + i m_{V_i}\Gamma_{V_i}} \\ & \times [3C_1(\mu) + C_2(\mu) + 3C_3(\mu) \\ & + C_4(\mu) + 3C_5(\mu) + C_6(\mu)]\,, \end{aligned} \tag{24}$$

where the phenomenological parameter κ is usually taken as ~ 2.3.

As for the values of the new Wilson coefficients, they are the free parameters in this work, but it is possible to establish the ranges out of the experimentally measured branching ratios of the semileptonic and also purely leptonic rare B meson decays:

$${\rm BR}(B \to K\ell^+\ell^-) = \left(0.75^{+0.25}_{-0.21} \pm 0.09\right)\times 10^{-6},$$
$${\rm BR}(B \to K^*\mu^+\mu^-) = \left(0.9^{+1.3}_{-0.9} \pm 0.1\right)\times 10^{-6},$$

reported by the Belle and Babar Collaborations [28, 29]. There is now also available an upper bound of pure leptonic rare B decays in the $B^0 \to \mu^+\mu^-$ mode [30]:

$${\rm BR}(B^0 \to \mu^+\mu^-) \le 2.0\times 10^{-7}.$$

Being in accordance with this upper limit and also with the above mentioned measurements of the branching ratios for the semileptonic rare B decays, we take in this work all new Wilson coefficients as real and varying in the region $-4 \le C_X \le 4$.

Among the new Wilson coefficients that appear in (1), those related to the helicity-flipped counter-parts of the SM operators, namely, C_{RL} and C_{RR}, vanish in all models with minimal flavor violation in the limit $m_s \to 0$. However, there are some MSSM scenarios in which there are finite contributions from these vector operators even for a vanishing s-quark mass. In addition, scalar type interactions can also contribute through the neutral Higgs diagrams in e.g. multi-Higgs doublet models and MSSM for some regions of the parameter spaces of the related models. In the literature, there exist studies to establish the ranges out of the constraints under various precision measurements for these coefficients (see e.g. [31]), and our choices for the range of the new Wilson coefficients are in agreement with these calculations.

To make some numerical predictions, we also need the explicit forms of the form factors A_0, A_+, A_-, V, a_0, a_+ and g. In our work we have used the results of [6], in which the q^2 dependencies of the form factors are given by

$$F(q^2) = \frac{F(0)}{(1 - as + bs^2)^2}\,,$$

where the values of the parameters $F(0)$, a and b for the $B_c \to D_s^*$ decay are listed in Table 2.

We present the results of our analysis in a series of figures. Before discussing these figures, we give our SM predictions for the longitudinal, transverse and the normal components of the lepton polarizations for $B_c \to D_s^*\ell^+\ell^-$ decay for the μ (τ) channel for reference:

$$\left\langle P_{\rm L}^-\right\rangle = 0.6211(0.6321)\,,$$
$$\left\langle P_{\rm T}^-\right\rangle = 0.0017(0.0468)\,,$$
$$\left\langle P_{\rm N}^-\right\rangle = -0.0837(-0.17)\,.$$

Table 1. Values of the SM Wilson coefficients at the $\mu \sim m_b$ scale

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	C_9	C_{10}
−0.248	+1.107	+0.011	−0.026	+0.007	−0.031	−0.313	+4.344	−4.624

Table 2. B_c meson decay form factors in a relativistic constituent quark model

	$F(0)$	a	b
A_0	0.279	1.30	0.149
A_+	0.156	2.16	1.15
A_-	−0.321	2.41	1.51
V	0.290	2.40	1.49
a_0	0.178	1.21	0.125
a_+	0.178	2.14	1.14
g	0.179	2.51	1.67

Figures 1 and 2 give the dependence of the integrated branching ratio (BR) on the new Wilson coefficients for the $B_c \to D_s^* \mu^+ \mu^-$ and $B_c \to D_s^* \tau^+ \tau^-$ decays, respectively. From these figures we see that BR depends strongly on the tensor interactions and weakly on the vector interactions, while it is completely insensitive to the scalar type of interactions. It is also clear from these figures that the dependence of the BR on the new Wilson coefficients is symmetric with respect to the zero point for the muon final state, but such a symmetry is not observed for the tau final state for the tensor interactions.

In Figs. 3 and 4, we present the dependence of the averaged longitudinal polarization $\langle P_L^- \rangle$ of ℓ^- and the combined averaged $\langle P_L^- + P_L^+ \rangle$ for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients. We observe that $\langle P_L^- \rangle$ is more sensitive to the existence of the tensor type interactions, while the combined average $\langle P_L^- + P_L^+ \rangle$ is sensitive to that of the scalar type interactions only. The fact that $\langle P_L^- + P_L^+ \rangle$ does not exhibit any dependence on the vector type of interactions is a result already expected, since the vector type interactions are cancelled when the longitudinal polarizations asymmetry of the lepton and antilepton are considered together. We also note that the values of $\langle P_L^- \rangle$ become substantially different from the SM value (at $C_X = 0$) as C_X becomes different from zero, which indicates that the measurement of the longitudinal lepton polarization in $B_c \to D_s^* \mu^+ \mu^-$ decay can be very useful to investigate new physics beyond the SM. From Fig. 3, we see that the contributions coming from all types of interactions to $\langle P_L^- \rangle$ are positive and they are increasing (decreasing) functions of both C_T and C_{TE} for negative (positive)

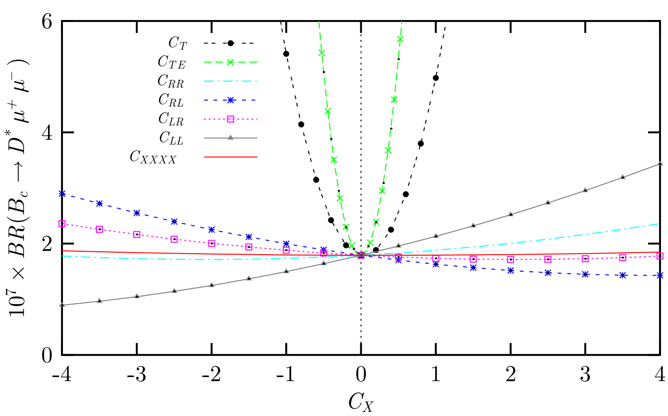

Fig. 1. The dependence of the integrated branching ratio for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

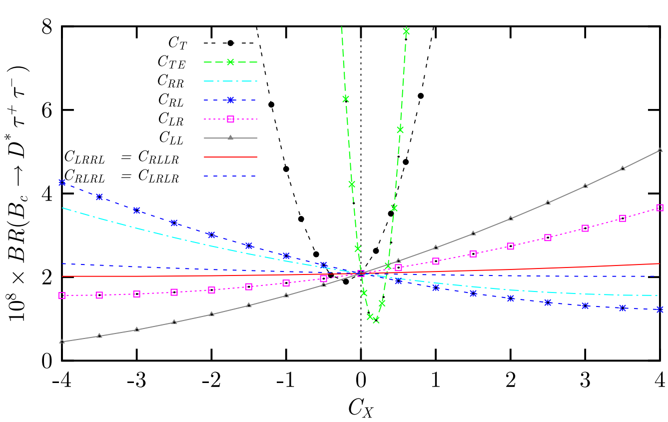

Fig. 2. The dependence of the integrated branching ratio for the $B_c \to D_s^* \tau^+ \tau^-$ decay on the new Wilson coefficients

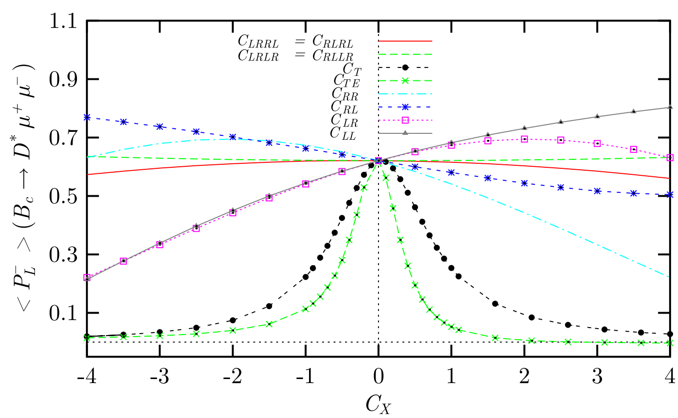

Fig. 3. The dependence of the averaged longitudinal polarization $\langle P_L^- \rangle$ of ℓ^- for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

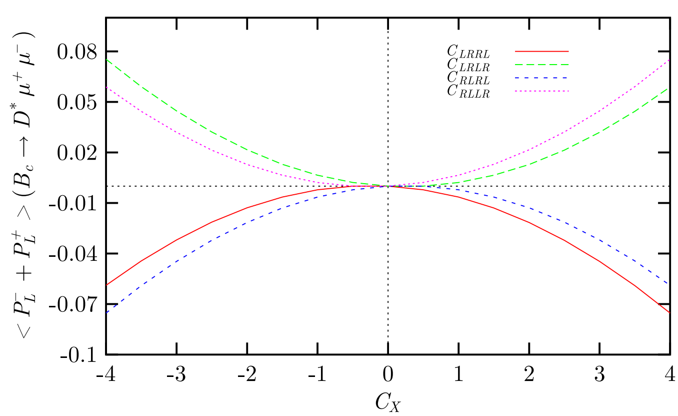

Fig. 4. The dependence of the combined averaged longitudinal lepton polarization $\langle P_L^- + P_L^+ \rangle$ for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

values. We observe from Fig. 4 that $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$ becomes zero at $C_X = 0$, which conforms to the SM results, and its dependence on C_X is symmetric with respect to this zero value. It is also interesting to note that $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$ is positive for all values of C_{LRLR} and C_{RLLR}, while it is negative for the remaining scalar type interactions.

Figures 5 and 6 are the same as Figs. 3 and 4, but for $B_c \to D_s^* \tau^+ \tau^-$. Similar to the muon case, $\langle P_{\rm L}^- \rangle$ is more sensitive to the tensor interactions than the other ones. The contributions to $\langle P_{\rm L}^- \rangle$ from all types of interactions are positive for all values of C_X except for C_{TE}: in the region $0.25 \lesssim C_{TE} < 4$, $\langle P_{\rm L}^- \rangle$ changes sign and becomes negative. As for the main interesting point in Fig. 6, although $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$ for the $B_c \to D_s^* \mu^+ \mu^-$ decay depends only on the scalar interactions, for $B_c \to D_s^* \tau^+ \tau^-$ decay it is also, and very sensitively, dependent on the tensor type of interactions. It is also interesting to note that $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$ changes sign: it takes positive (negative) values for the negative (positive) values of C_T and C_{TE}. Thus, one can provide valuable information about new physics by determining the sign and the magnitude of $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$. We finally note that as in the case of the muon final state, in the tau final state too, $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$ becomes zero at $C_X = 0$ and confirms the SM result.

In Figs. 7 and 8, we present the dependence of the averaged transverse polarization $\langle P_{\rm T}^- \rangle$ of ℓ^- and the combined

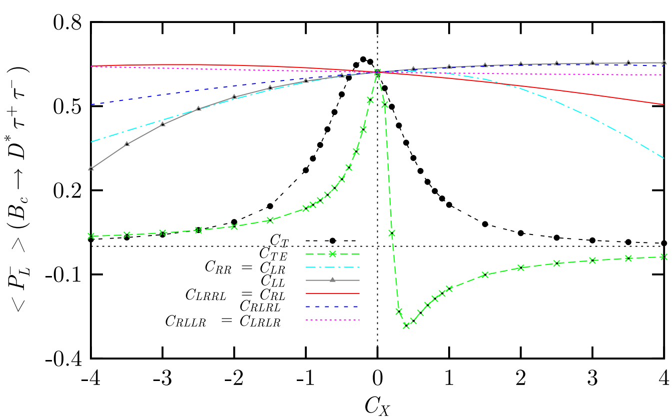

Fig. 5. The same as Fig. 3, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

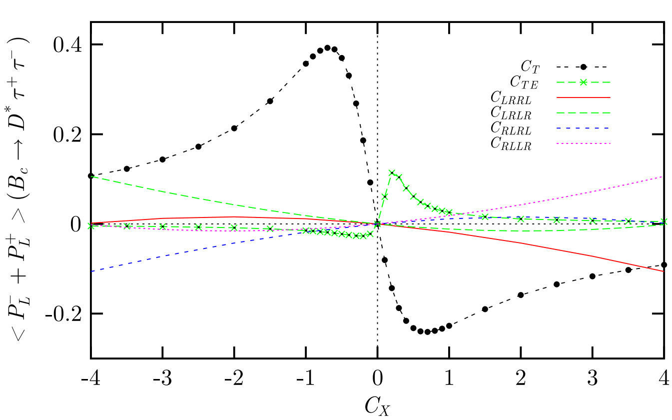

Fig. 6. The same as Fig. 4, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

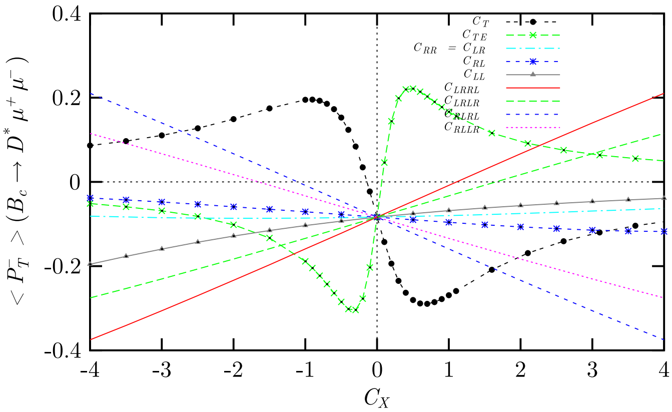

Fig. 7. The dependence of the averaged transverse polarization $\langle P_{\rm T}^- \rangle$ of ℓ^- for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

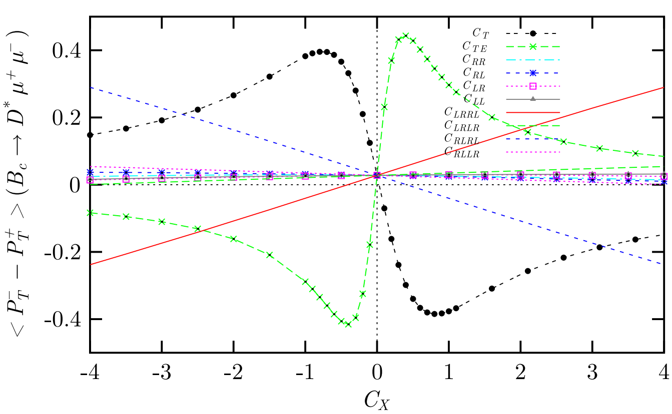

Fig. 8. The dependence of the combined averaged transverse lepton polarization $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ for the $B_c \to D_s^* \gamma \mu^+ \mu^-$ decay on the new Wilson coefficients

averaged $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ for $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients. From these figures, it is seen that for $\langle P_{\rm T}^- \rangle$, there appears a strong dependence on the tensor and scalar interactions and also a weak dependence on the vector interactions. On the other hand, the vector contributions to the $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ is negligible and the main contribution comes from the tensor interactions and the C_{LRRL} and C_{RLRL} components of the scalar interactions. As seen from Figs. 7 and 8, both $\langle P_{\rm T}^- \rangle$ and $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ are positive (negative) for negative (positive) values of C_T and C_{RLRL}, except in a region about the zero values of the coefficients, $-1 \lesssim C_X \lesssim 1$, while their behaviors with respect to C_{TE} and C_{LRRL} are opposite. Therefore, determination of the sign and magnitude of these observables can also give useful information about the existence of new physics.

Figures 9 and 10 are the same as Figs. 7 and 8, but for $B_c \to D_s^* \tau^+ \tau^-$. We see from Fig. 9 that $\langle P_{\rm T}^- \rangle$ is quite sensitive to all types of interactions, and the behavior of the scalar interaction is identical for the coefficients C_{LRRL}, C_{LRLR} and C_{RLLR}, C_{RLRL} in pairs. It can be seen from Fig. 10 that, although the tensor and scalar interactions are dominant for $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$, the dependence of the vector interactions is also more sizable as compared with

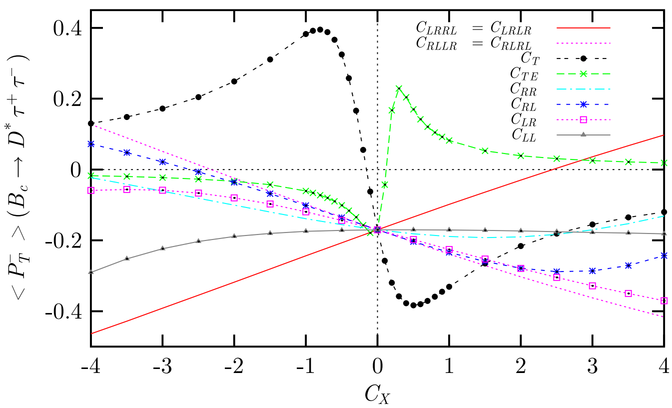

Fig. 9. The same as Fig. 7, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

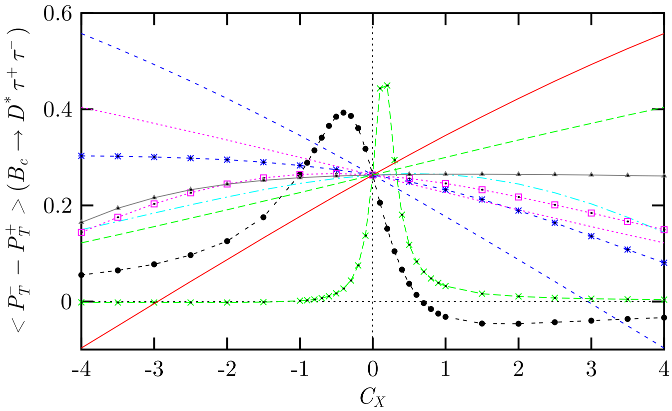

Fig. 10. The same as Fig. 8, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

the case of the muon final state. In addition, changes in sign of $\langle P_{\rm T}^- \rangle$ and $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ are observed depending on the change in the tensor and scalar interaction coefficients, whose measure may provide useful tools for new physics.

In Figs. 11 and 12, we present the dependence of the averaged normal polarization $\langle P_{\rm N}^- \rangle$ of ℓ^- and the combined averaged $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ for $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients. We see from Fig. 11 that $\langle P_{\rm N}^- \rangle$ strongly depends on the tensor interactions. Its dependence on the scalar type of interactions is moderate and identical for the coefficients C_{LRRL}, C_{LRLR} and C_{RLLR}, C_{RLRL} in pairs. As seen from Fig. 12, the behavior of $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ is determined by the tensor interactions only. We also observe that $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ is positive (negative) when $C_T < 0$ ($C_T > 0$),

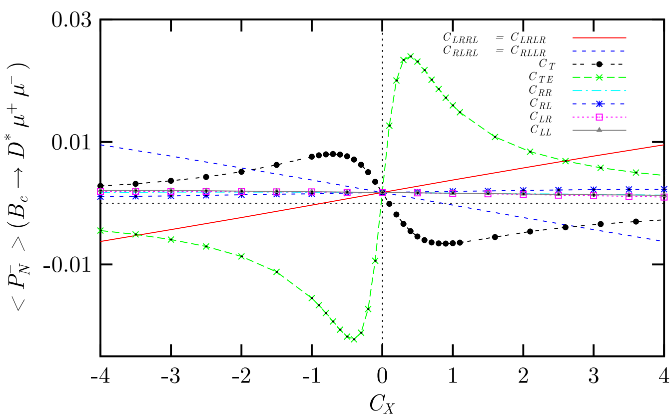

Fig. 11. The dependence of the averaged normal polarization $\langle P_{\rm N}^- \rangle$ of ℓ^- for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

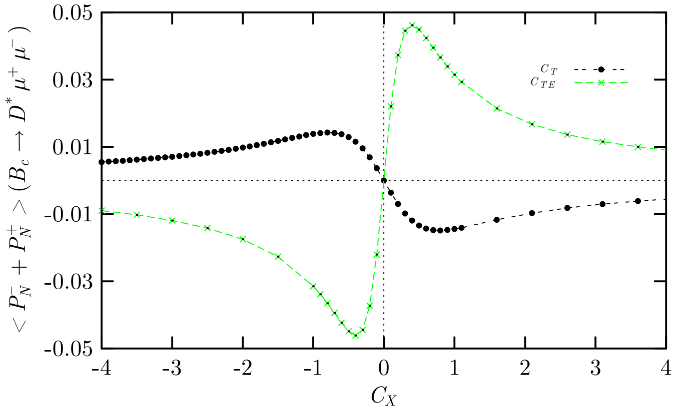

Fig. 12. The dependence of the combined averaged normal lepton polarization $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ for the $B_c \to D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients

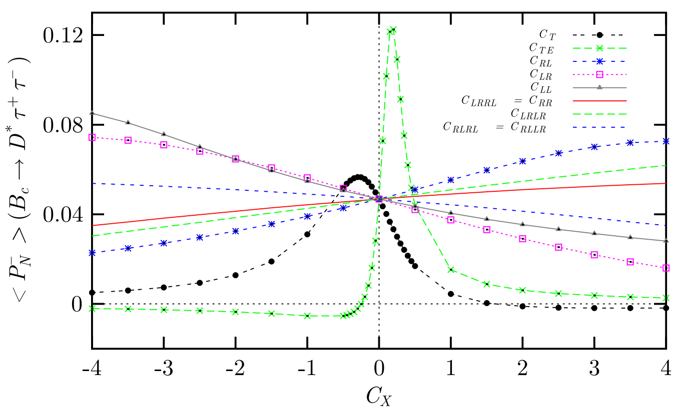

Fig. 13. The same as Fig. 10, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

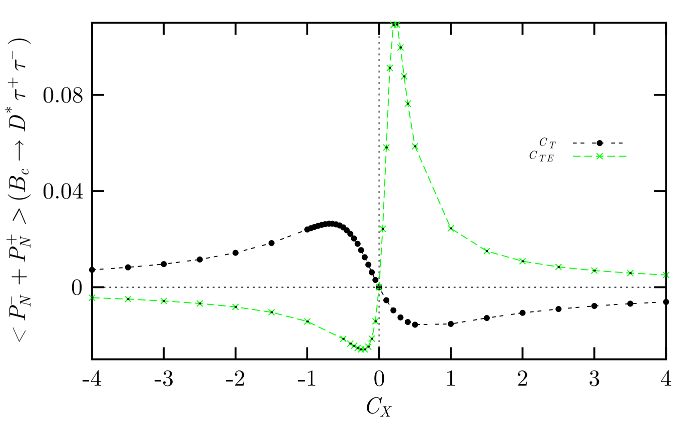

Fig. 14. The same as Fig. 11, but for the $B_c \to D_s^* \tau^+ \tau^-$ decay

while its behavior with respect to C_{TE} is opposite. Furthermore, $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ becomes zero at $C_X = 0$, as expected in the SM.

Figures 13 and 14 are the same as Figs. 11 and 12, but for $B_c \to D_s^* \tau^+ \tau^-$. We first note that, opposite to the muon final state case, here $\langle P_{\rm N}^- \rangle$ depends on all types of interactions, although the dependence on the tensor interaction is stronger. We also observe that $\langle P_{\rm N}^- \rangle$ always takes positive values, except when $C_{TE} \lesssim -0.25$ and $C_T \gtrsim 2$. As seen from Fig. 14, $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ depends only on the tensor interactions, and its behavior is the same as that of the muon final state case.

In conclusion, we present the most general analysis of the lepton polarization asymmetries in the rare $B_c \to D_s^* \ell^+ \ell^-$ decay using the general, model independent form of the effective Hamiltonian. The dependence of the longitudinal, transversal and normal polarization asymmetries of ℓ^- and their combined asymmetries on the new Wilson coefficients are studied. It is found that the lepton polarization asymmetries are very sensitive to the existence of tensor and scalar type interactions. Moreover, $\langle P_{\rm T} \rangle$ and $\langle P_{\rm N} \rangle$ change signs as the new Wilson coefficients vary in the region of interest. This conclusion is valid also for the combined polarization effects $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle$, $\langle P_{\rm T}^- - P_{\rm T}^+ \rangle$ and $\langle P_{\rm N}^- + P_{\rm N}^+ \rangle$ for the same decay channel. It is well known that in the SM $\langle P_{\rm L}^- + P_{\rm L}^+ \rangle = \langle P_{\rm T}^- - P_{\rm T}^+ \rangle = \langle P_{\rm N}^- + P_{\rm N}^+ \rangle \simeq 0$ in the limit $m_\ell \to 0$. Therefore any deviation from this relation and the determination of the sign of the polarization is decisive and is an effective tool in looking for new physics beyond the SM.

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